Fiat-Shamir for Proofs Lacks a Proof Even in the Presence of Shared Entanglement

QCrypt 2023

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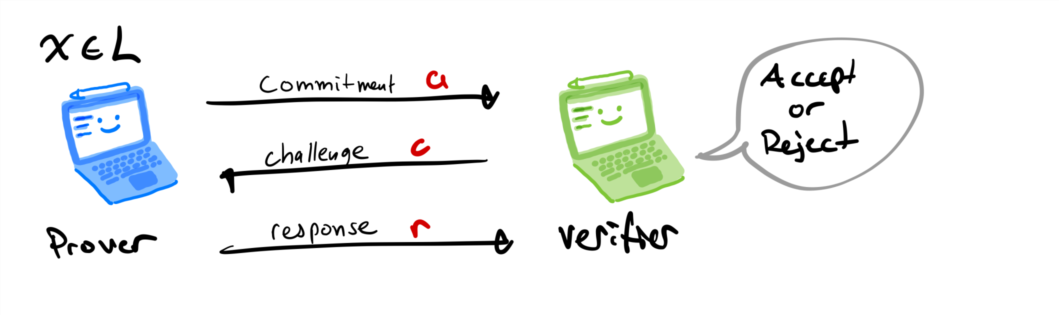
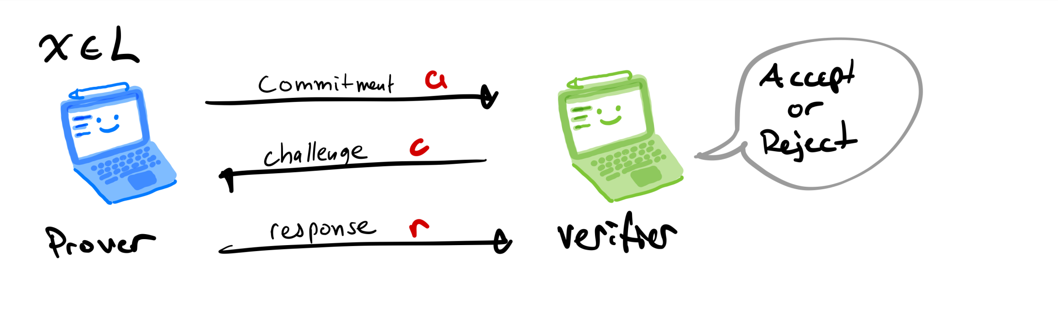
Σ–Protocols


x ∈ L
Commitment a
challenge c
response r
Prover
Verifier
Accept or Reject

Σ–Protocols



Public coins

c uniform in $\{0,1\}^m$

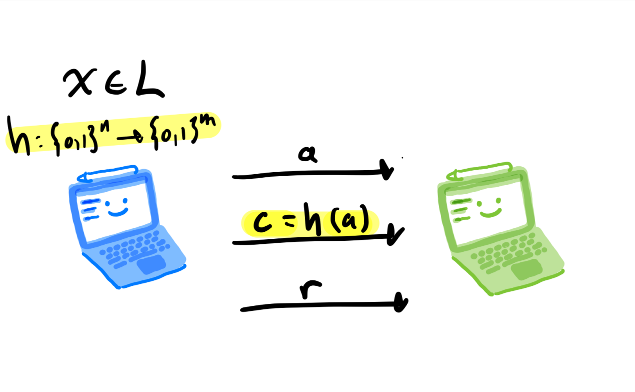
Special Soundness

If $x \notin L$, $\Pr_c[\text{accept}] = \frac{1}{2^m}$

Correctness

If $x \in L$, accept

Fiat-Shamir Transform



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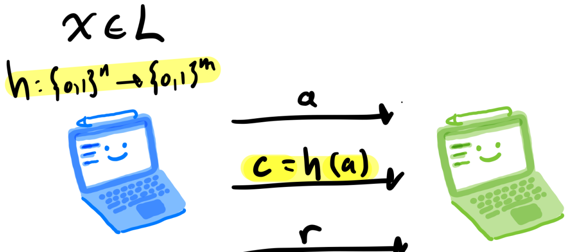
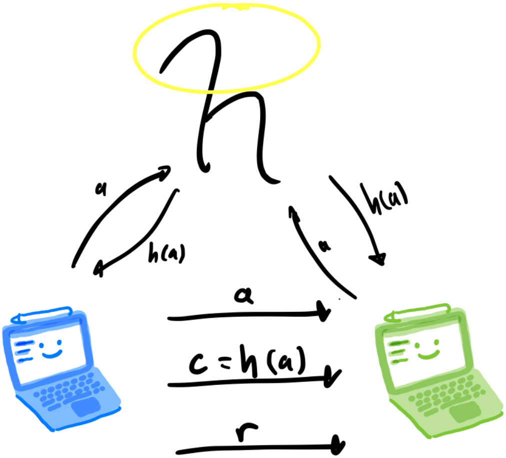


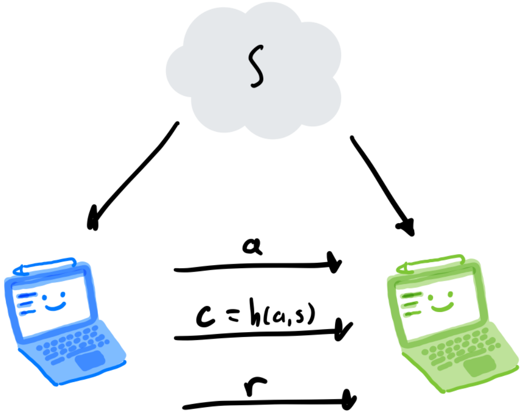
Universal: preserves soundness for all Σ–protocols

$h(a)$ should be unpredictable (random and independent of a)

In the Random Oracle Model



In the Common Reference String Model



Brief History of Fiat-Shamir Soundness

- Soundness is preserved in ROM & QROM[1]

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- CRS: unsound for proof[3]. There are proofs such that the security of FS cannot be shown by black-box reduction to a standard assumption.
- Positive results for *non-universal* FS in the CRS model.

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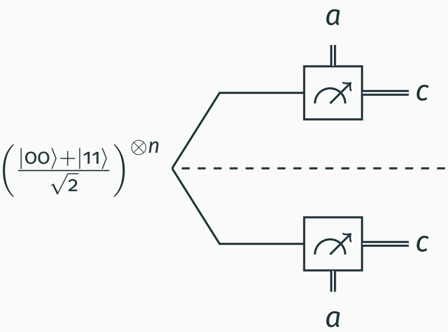
Can we have universality in the quantum world?

Quantum Entanglement as a Random Oracle?



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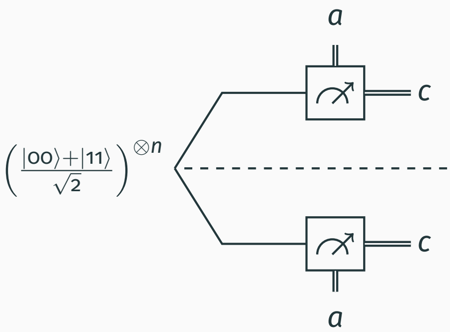
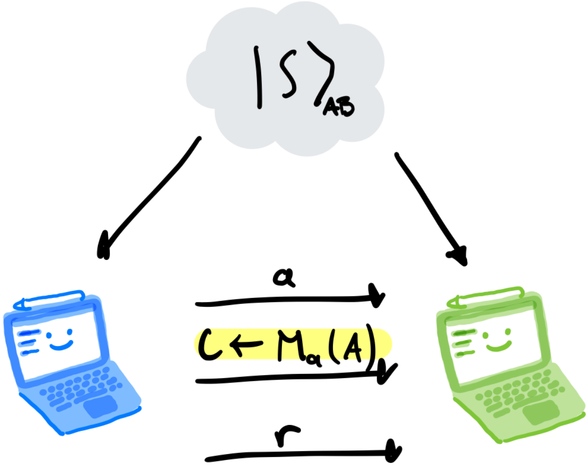


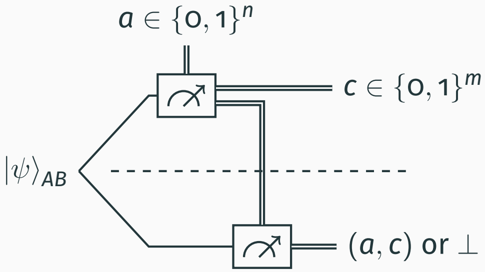
Oracle-like properties

- Uniformity: both get same random c
- Independence: mutually unbiased bases

The Common Reference Quantum State Model



A Weak One-Time Random Oracle



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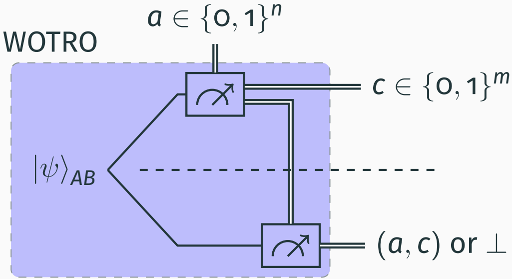

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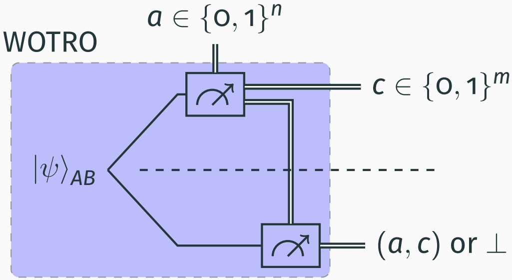

Security (δ–Avoiding)

For any $f : \{0,1\}^n \to \{0,1\}^m$,

$$\Pr[c = f(a)] \leq 1 - \delta$$

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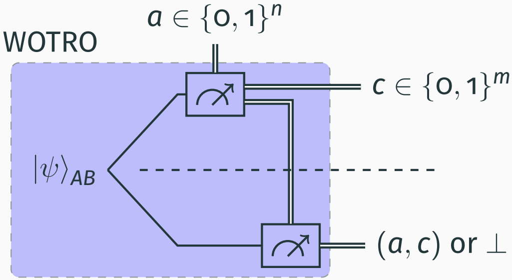

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$\implies$ Fiat-Shamir for Σ–protocols

Avoids *bad challenge* function of special sound proofs.

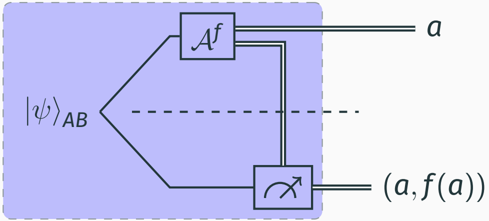
Theorem

There is no non-interactive WOTRO protocol using pre-shared entanglement that avoids every $f : \{0,1\}^n \to \{0,1\}^m$.

Proof sketch

- $\mathcal{A}^f$ hits a random function $f : \{0,1\}^n \to \{0,1\}^m$.

WOTRO



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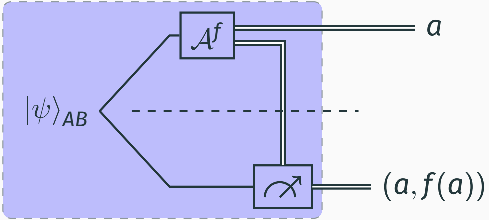

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- Goal: observe $N^a_{f(a)}$

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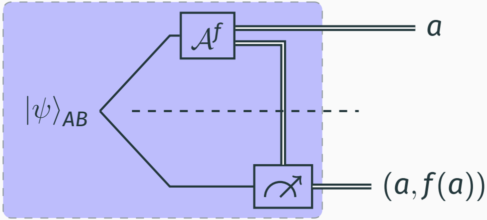

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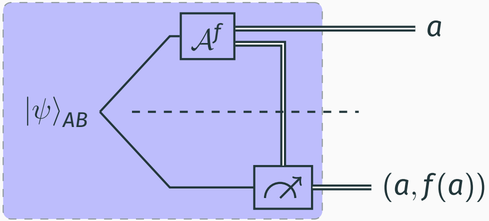

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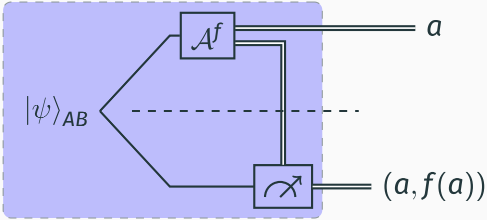

By Ahlswede and Winter's operator Chernoff bound,

$$\mathbb{E}_f[N^a_{f(a)}] = \frac{1}{2^m}\mathbb{I} \implies \Pr_f\left[\frac{1}{2^n}\sum_{a\in\{0,1\}^n} N^a_{f(a)} \not\leq (1+\eta)\frac{1}{2^m}\mathbb{I}\right] \leq \mathrm{negl}(n-m)$$

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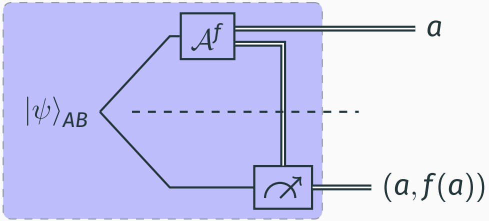
WOTRO



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This means that $\left\{\frac{2^m}{2^n(1+\eta)} N_{f(a)}^a\right\}_a$ (almost) forms a POVM.

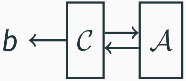
What about computational security?

Theorem

There is no non-interactive WOTRO protocol using pre-shared entanglement whose security can be proven from a (1) **cryptographic game assumption** using a (2) **fully black-box reduction**.

1 Cryptographic Games

A *cryptographic game assumption* $\mathcal{G} = (\mathcal{C}, p)$ is composed of a challenger $\mathcal{C}$ and a probability p.



Game is secure if for any efficient $\mathcal{A}$, $\Pr[b = 1] \leq p + \text{negl}(n)$

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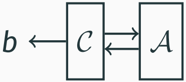

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Search games $(p = 0)$

- LWE
- preimage resistance
- collision resistance
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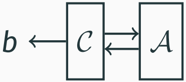

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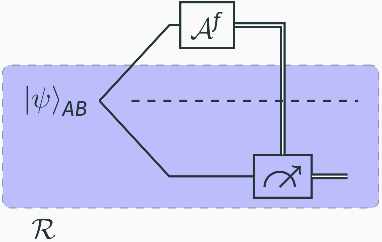
Guessing games $(p = \frac{1}{2})$

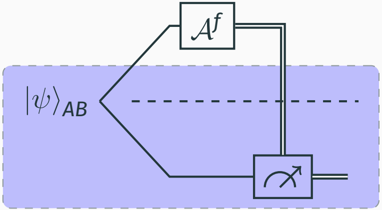
- DLWE
- IND-CCA
- pseudorandomness

Reductions from WOTRO...



Fully Black-Box Reductions

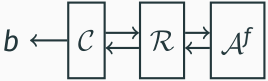
Reductions from WOTRO...



$\mathcal{R}$

...to cryptographic game $(\mathcal{C}, p)$

$\mathcal{R}$ plays the game with $\mathcal{C}$ and has input/output access to $\mathcal{A}^f$



If adversary $\mathcal{A}^f$ wins with probability $\frac{1}{\text{poly}(n)}$,

$$\Pr[b = 1] \geq p + \frac{1}{\text{poly}(n)}$$

Proof sketch

Simulation

Adversary $\{\mathcal{A}^f\}_f$ is *simulatable*: $\exists$ Sim $\forall$ PPT $\mathcal{D}$,

$$\langle \mathcal{D} \rightleftharpoons \mathcal{A}^f \rangle \approx \langle \mathcal{D} \rightleftharpoons \mathsf{Sim} \rangle$$

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If $\mathcal{R}^{\mathcal{A}^f}$ breaks game $\mathcal{G}$, then $\mathcal{R}^{\mathsf{Sim}}$ also breaks game $\mathcal{G}$, but efficiently.

Other results

Applications of WOTRO impossibility

- Universal Fiat-Shamir is black-box impossible in the CRQS model
- Tasks that imply WOTRO are impossible, e.g. strenghtening of quantum lightning

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Non-game assumption for universal Quantum Fiat-Shamir

Secure quantum protocol based on the hardness of producing a superposition of many collisions over many hash functions. (Classical: based on subexp obfuscation & OWF[4])

[4] Kalai, G. N. Rothblum, and R. D. Rothblum, “From Obfuscation to the Security of Fiat-Shamir for Proofs”.

Thank you!