Tutorial Talk: Certified Deletion

James Bartusek

UC Berkeley

Outline

1. Basic scenario and applications
2. Recipe for constructions
3. Security
4. Certifiable deletion of programs

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Certified Deletion: Cloud Storage

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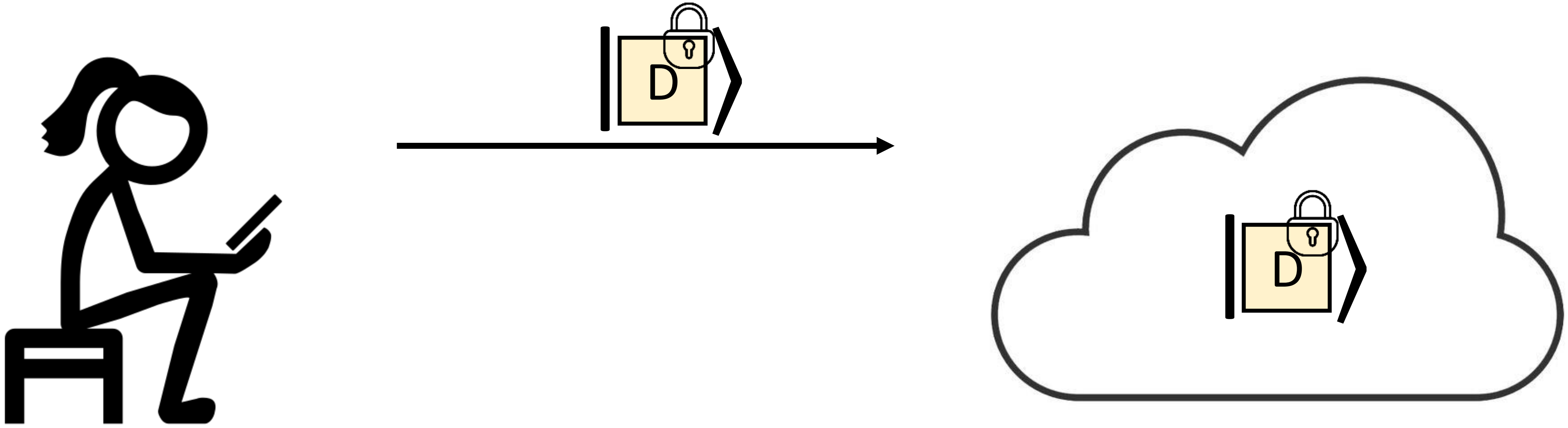

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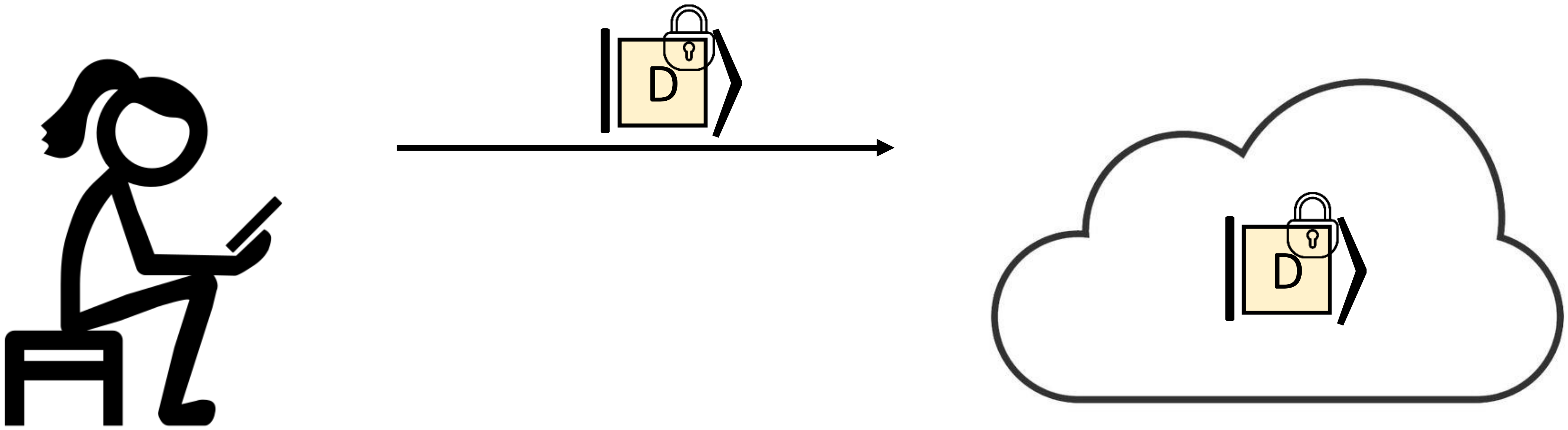

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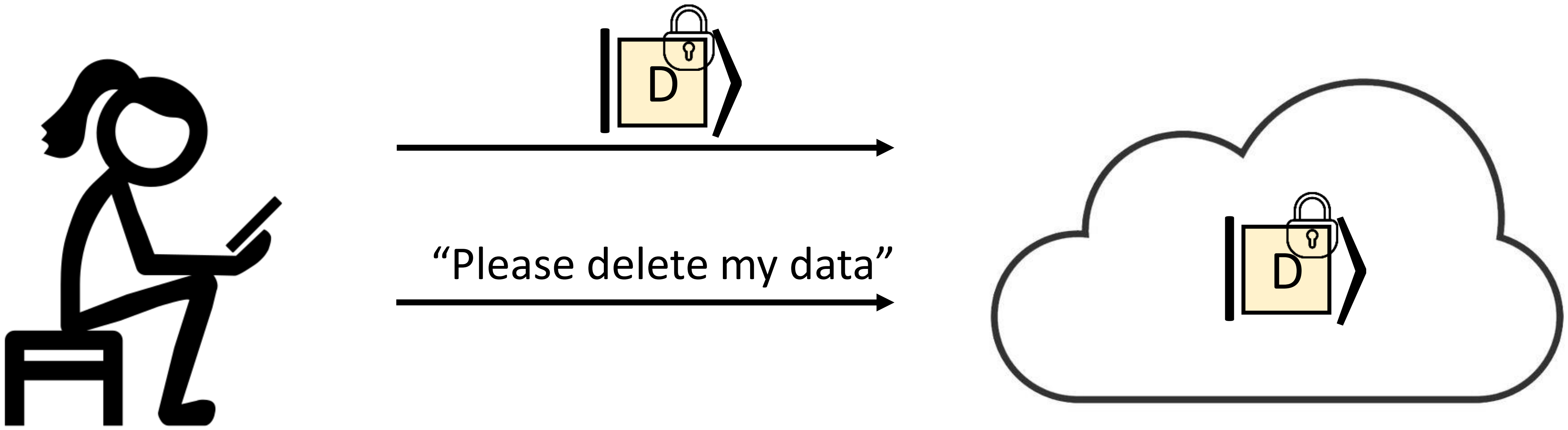

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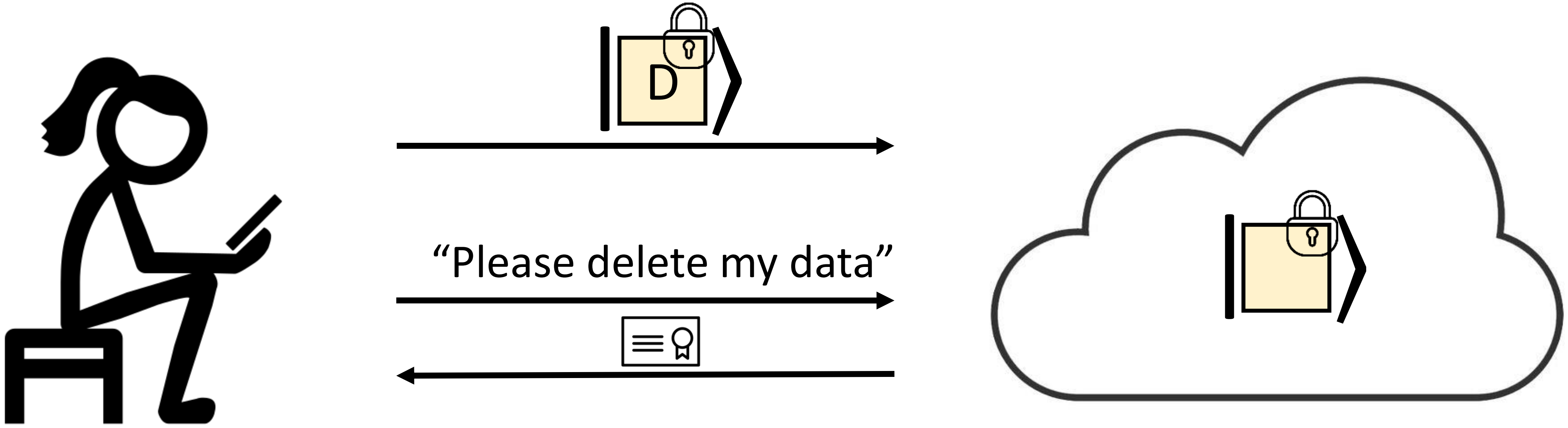

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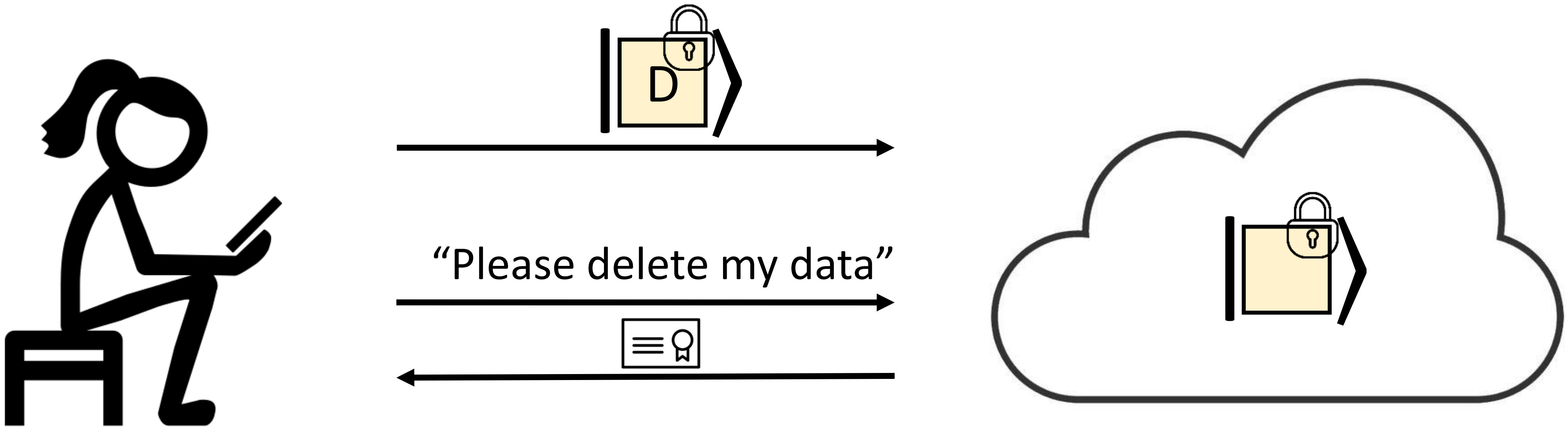

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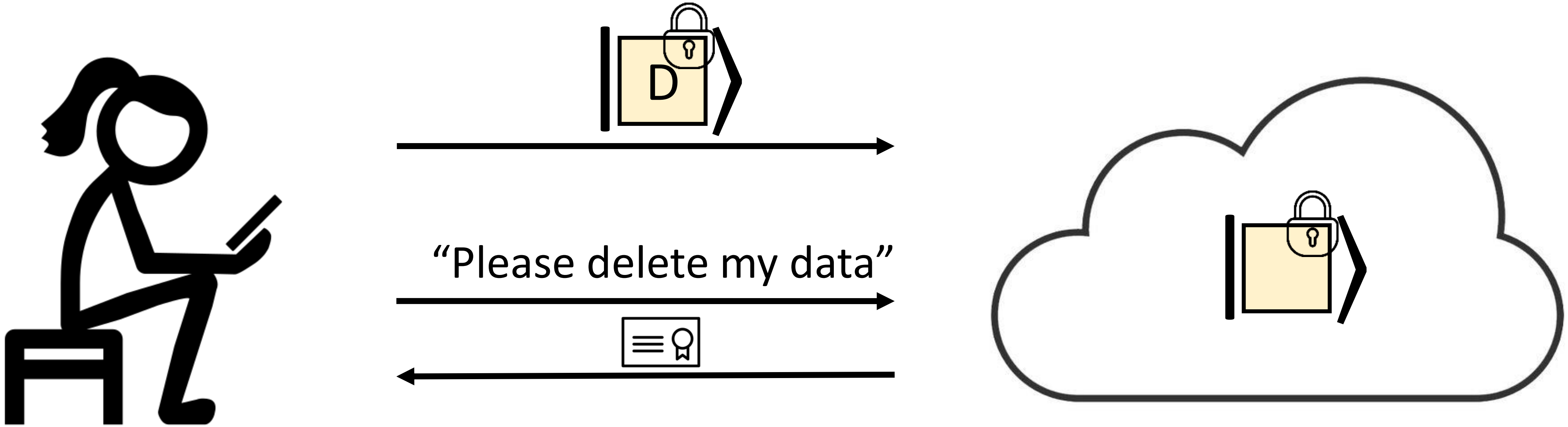

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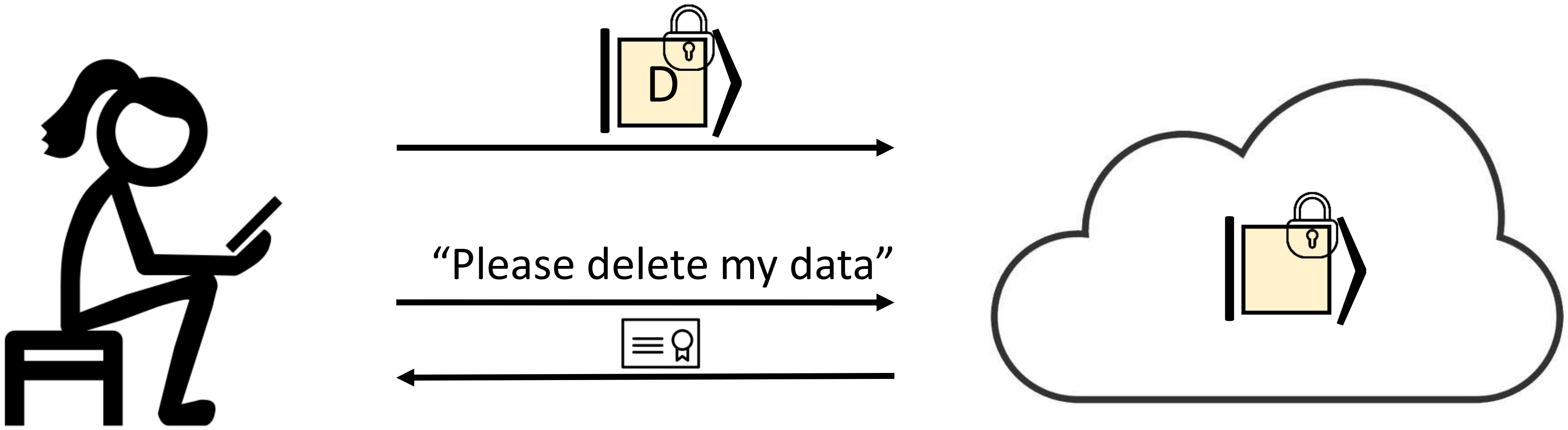

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[Broadbent, Islam 20]
[Hiroka, Morimae, Nishimaki, Yamakawa 21]

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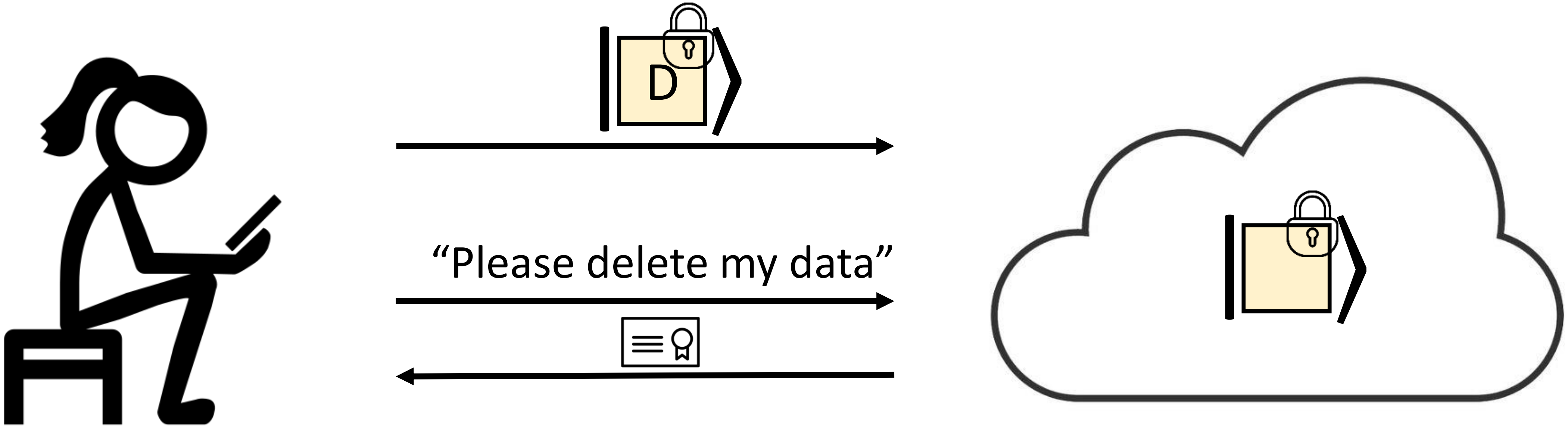

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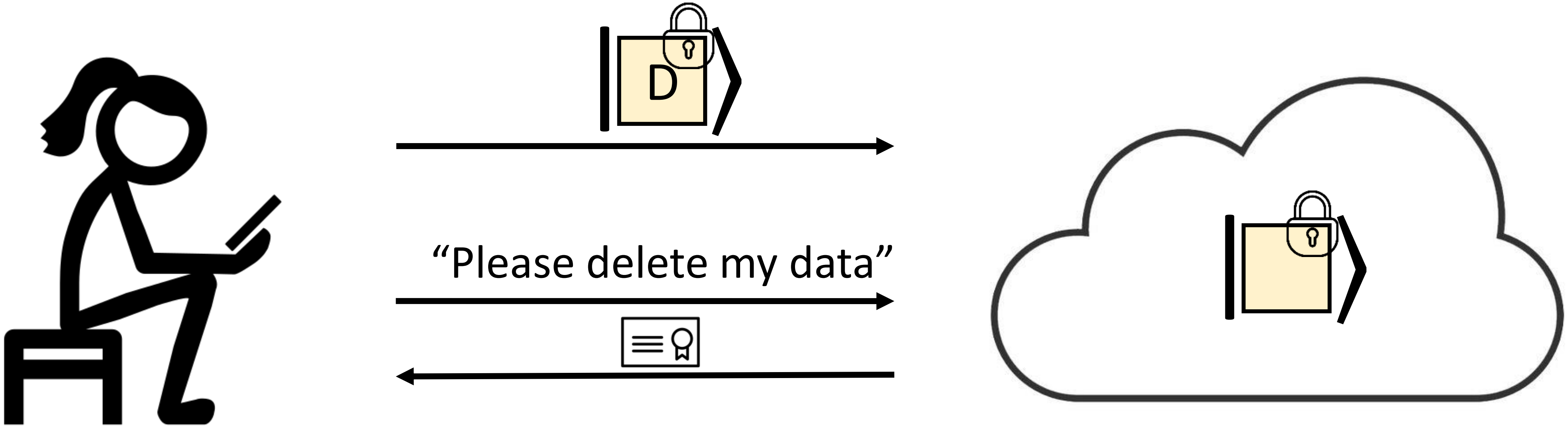
Classically: [Garg, Goldwasser, Vasudevan 20]



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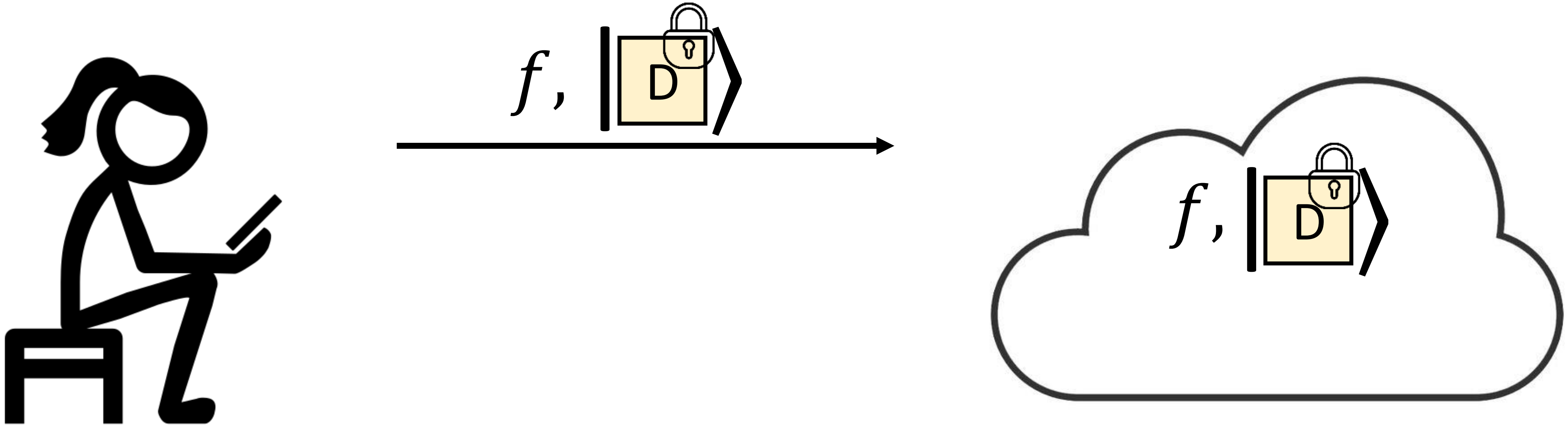

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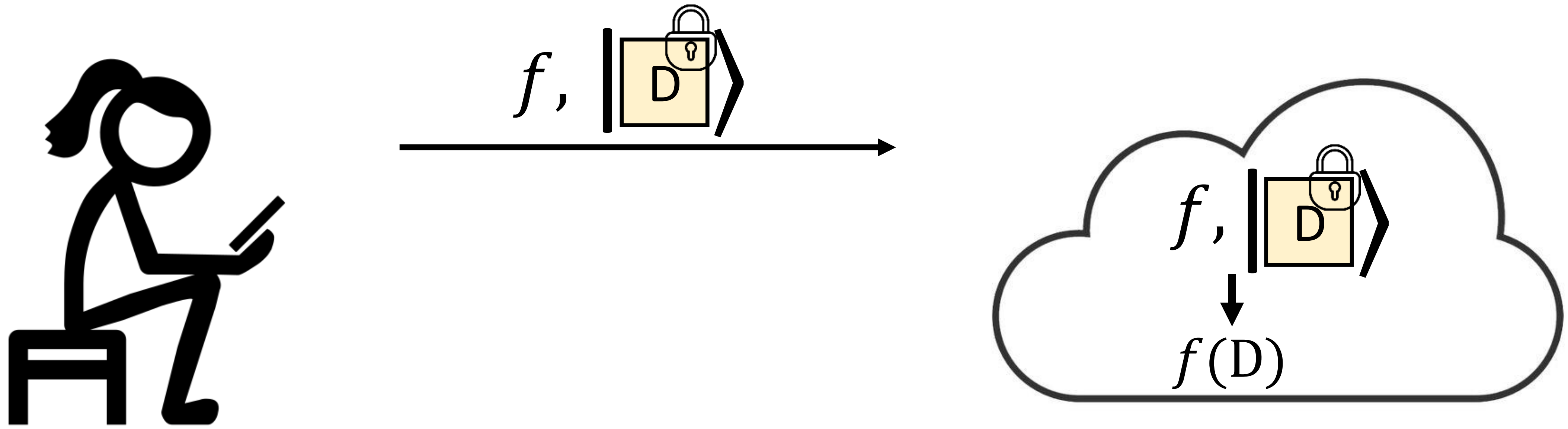

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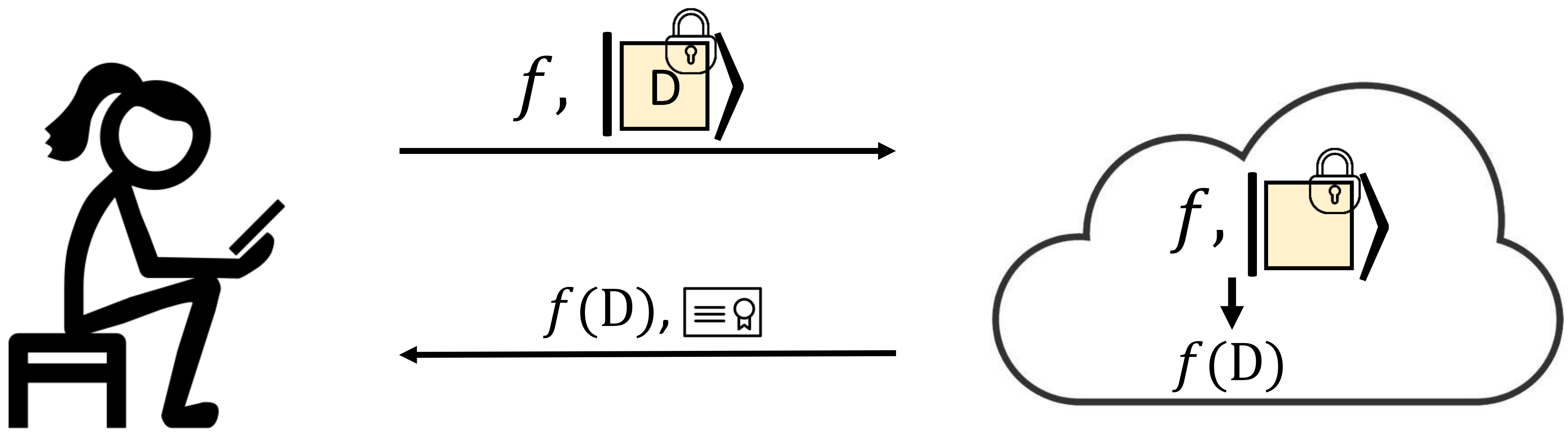

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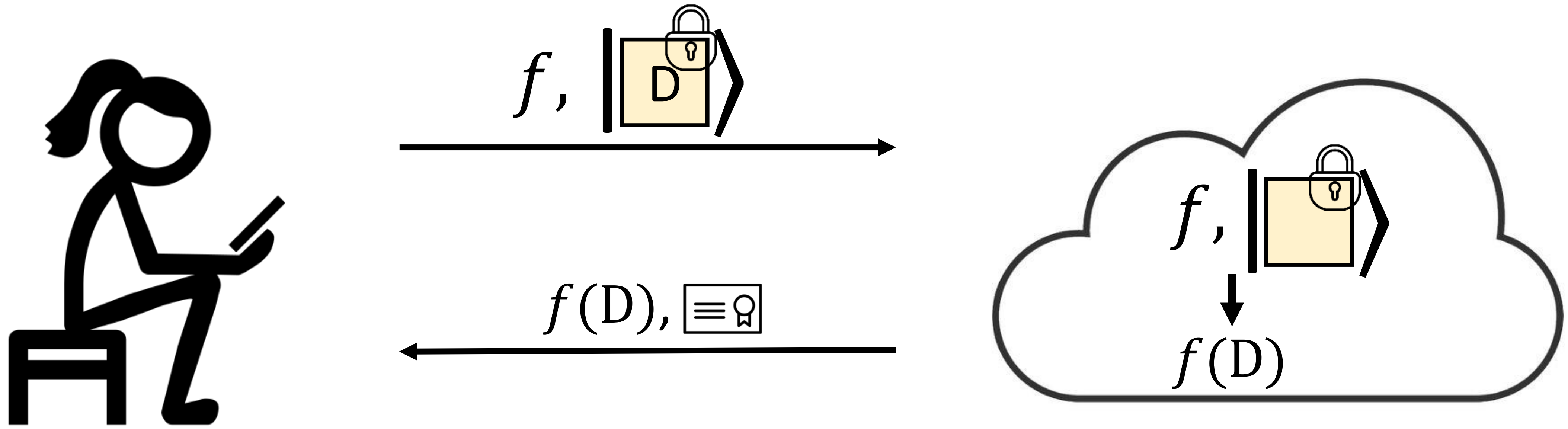

- Server can compute and return $f(\mathrm{D})$ along with a proof [certificate icon] that they erased *all other* information about D

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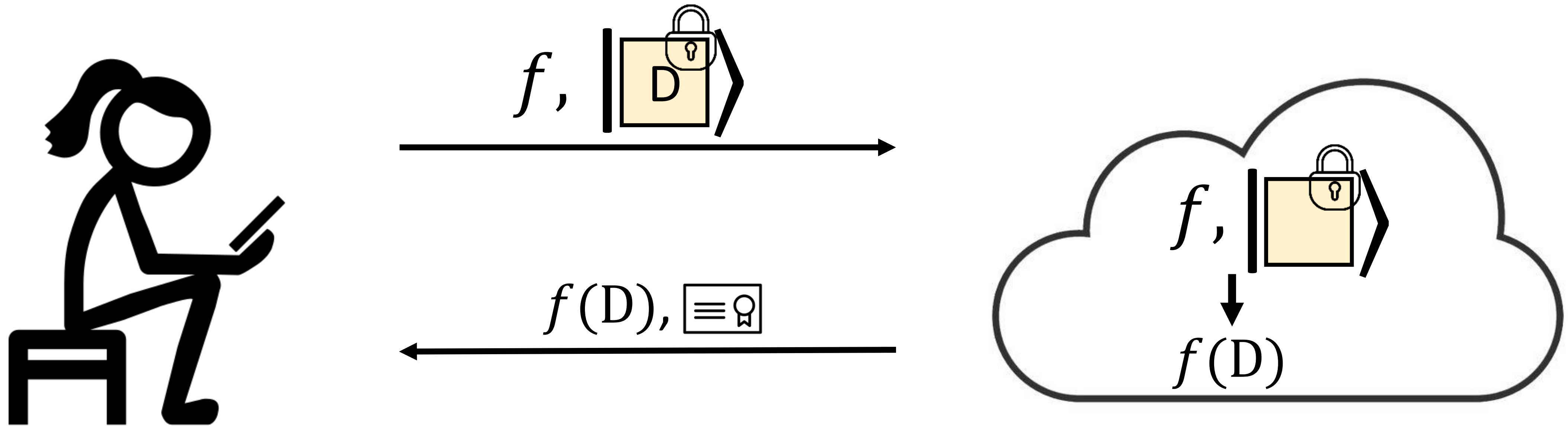
[Broadbent, Islam 20]
[Poremba 23]
[**B**, Garg, Goyal, Khurana, Malavolta, Raizes, Roberts 23]



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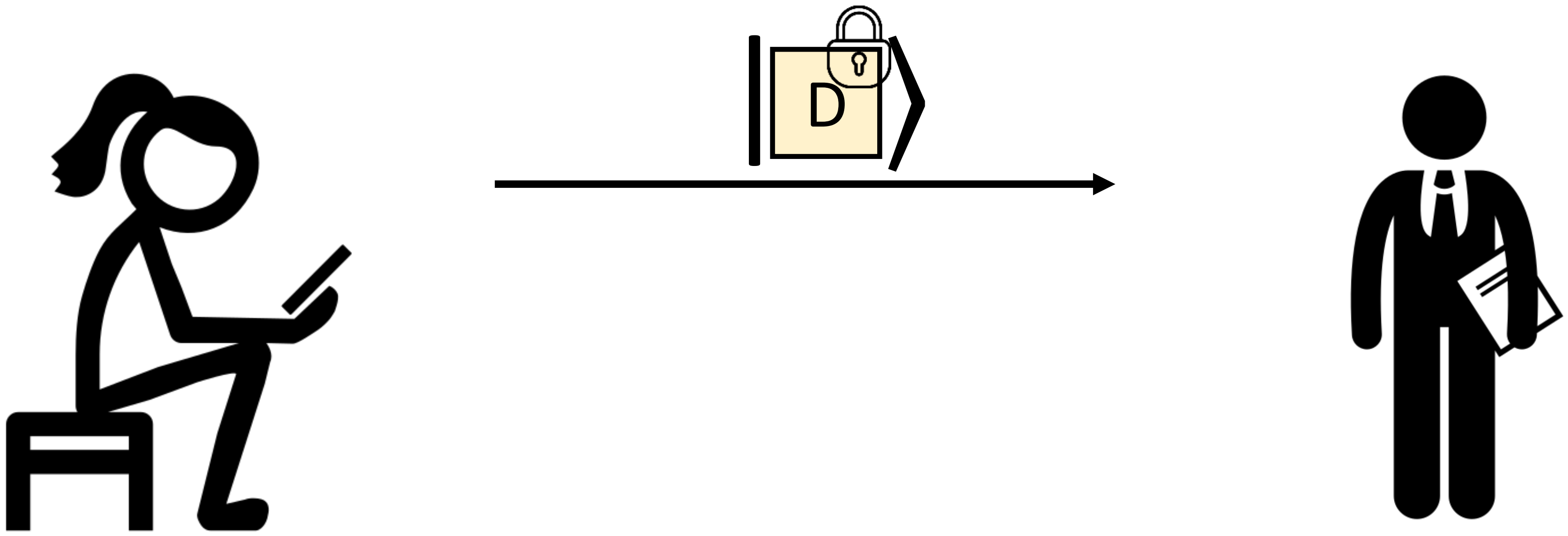

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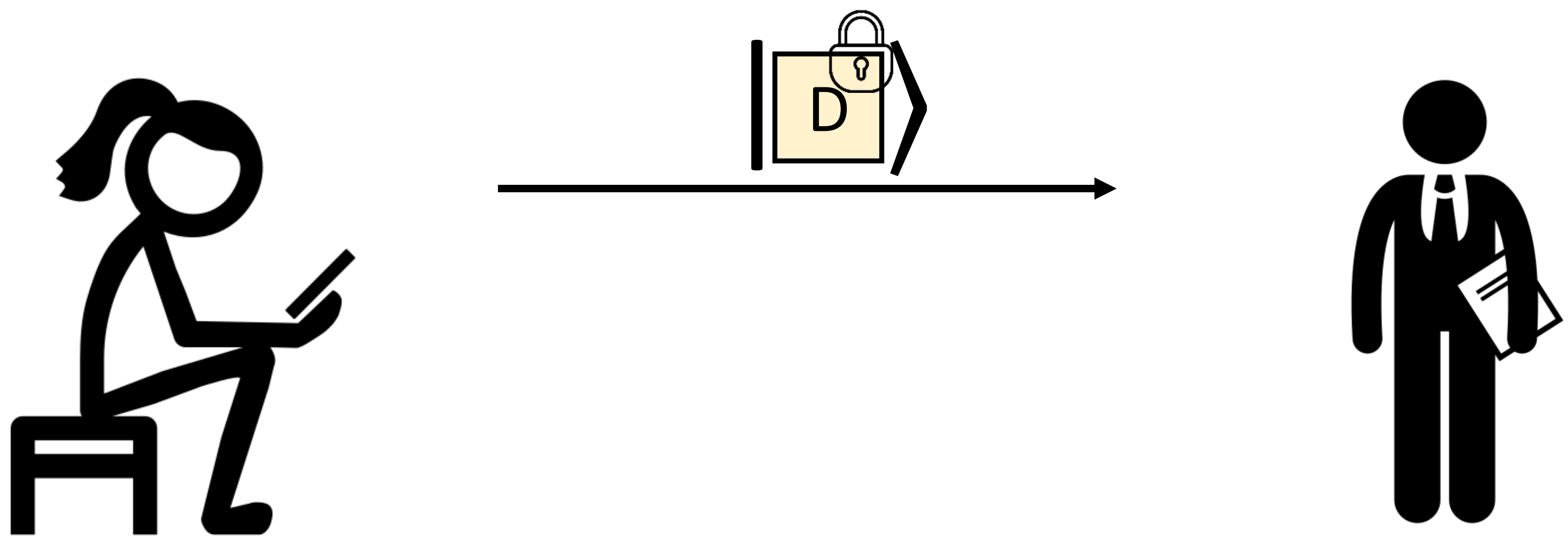
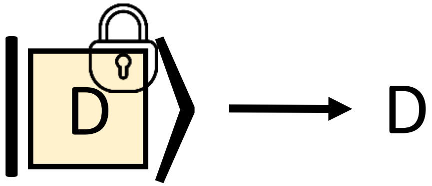


After time T:



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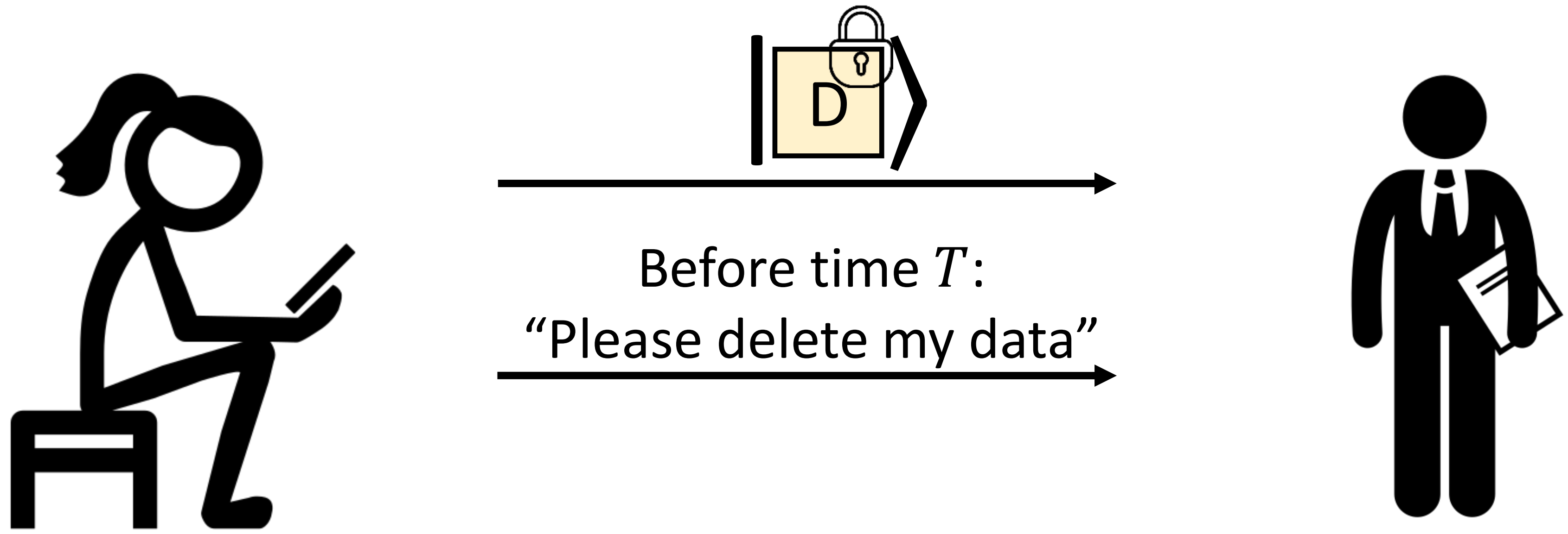

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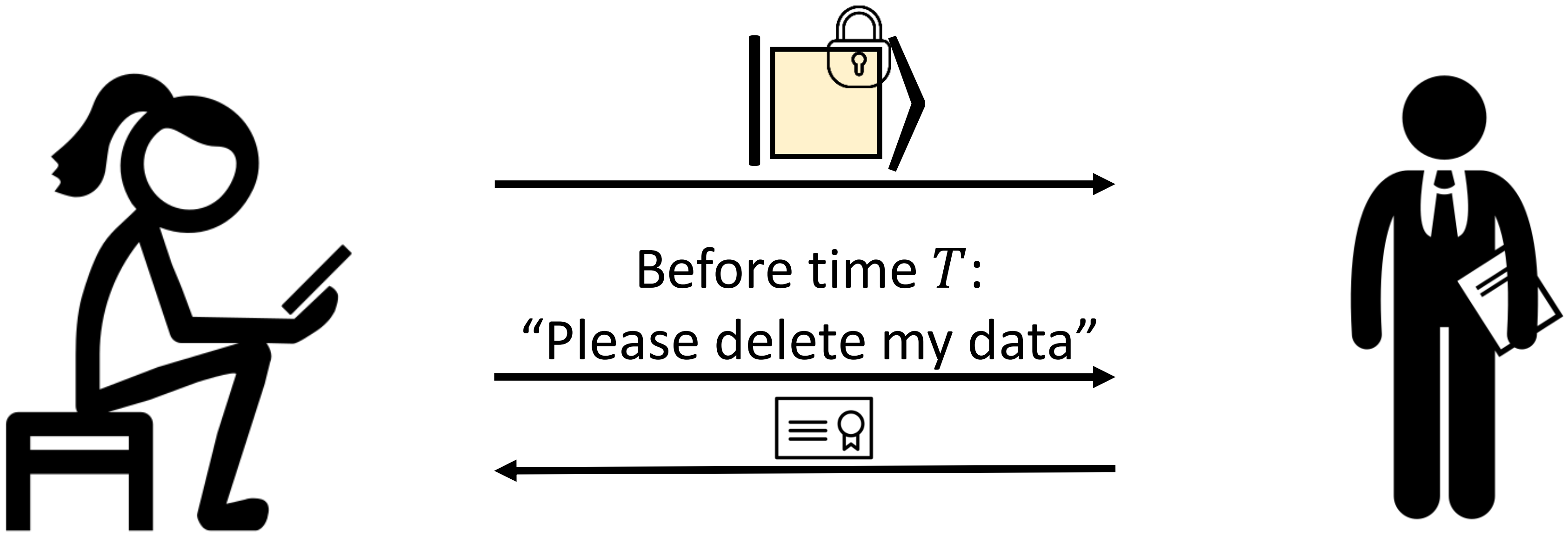

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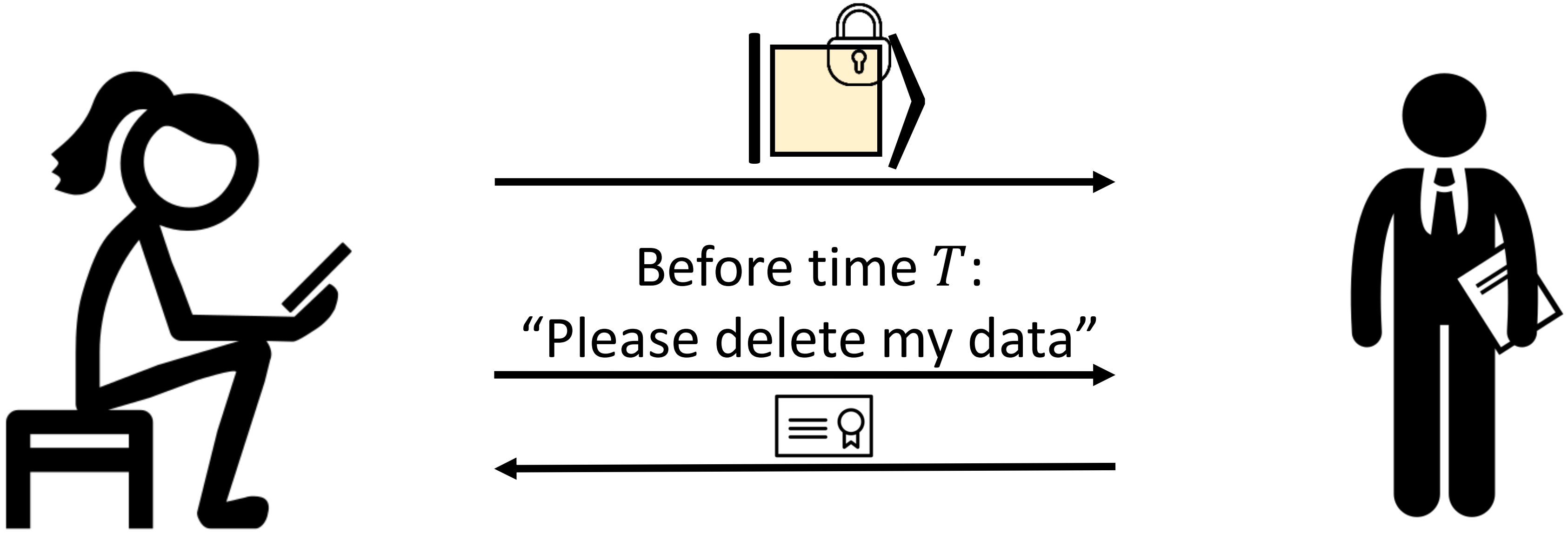

- Wills

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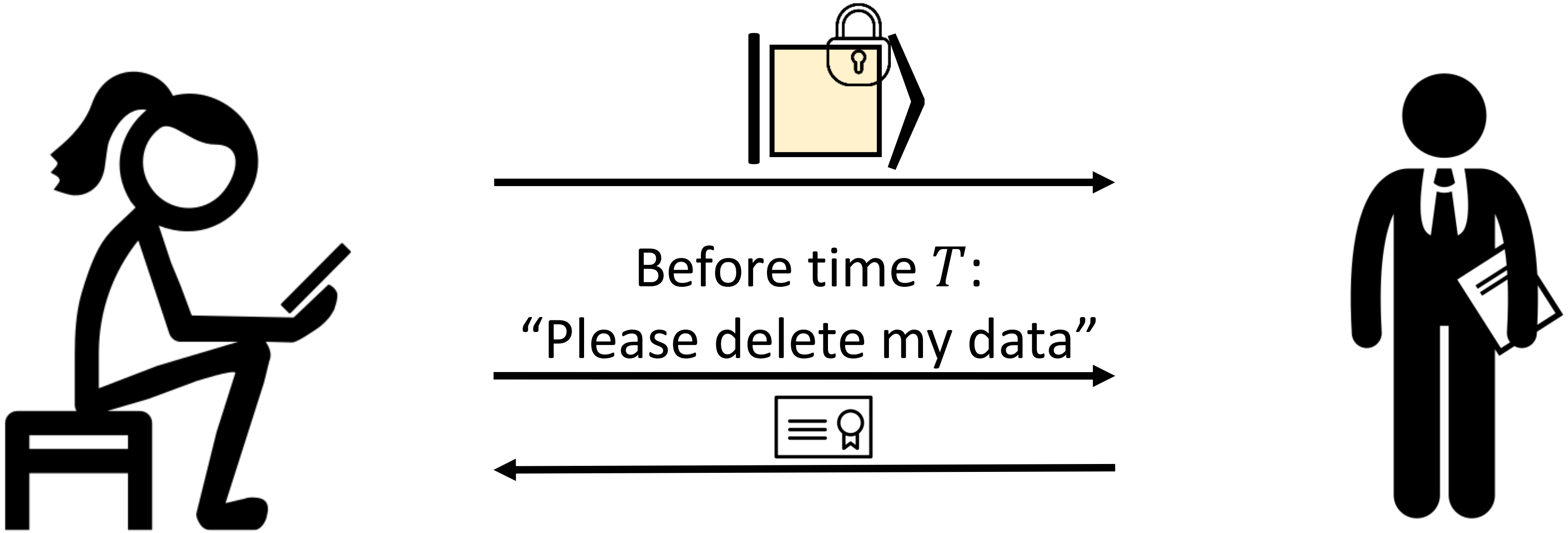
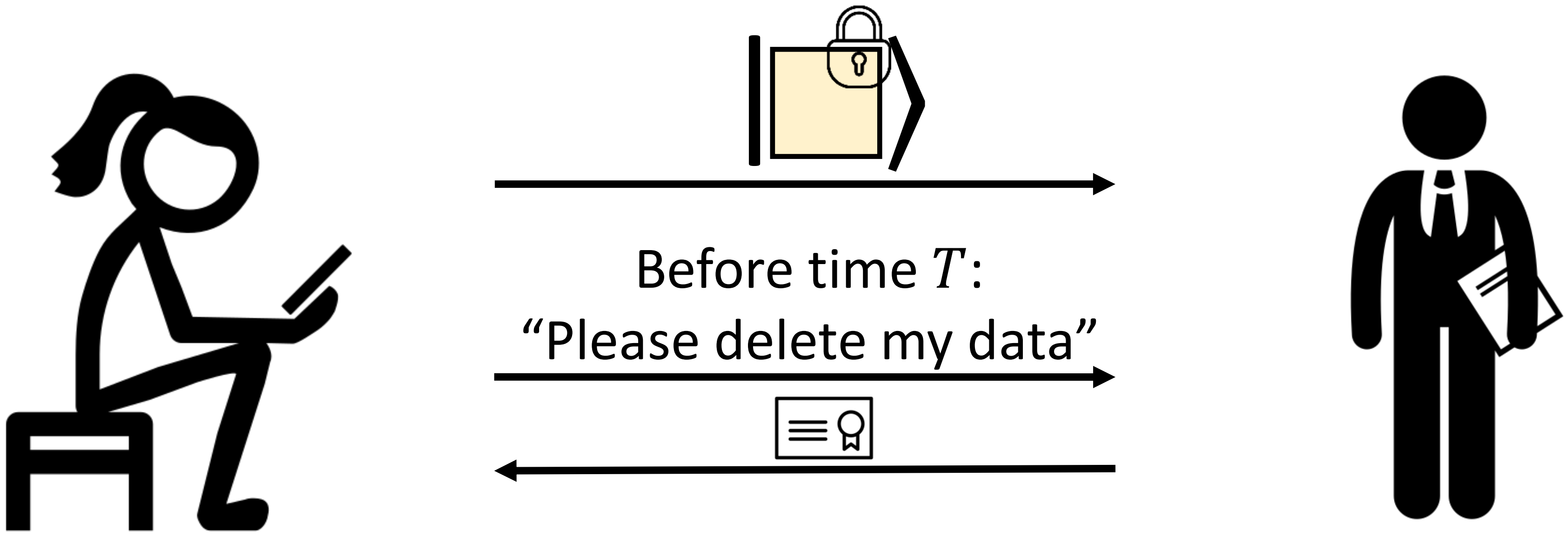


- Wills
- Deposits

Certified Deletion: Timed-Release Encryption

[Unruh 13]
[**B**, Khurana 23]



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- Take advantage of the uncertainty principle
- We need states that can simultaneously encode information in two conjugate bases
  - One basis will encode plaintext information
  - The other will encode valid deletion certificates

General Recipe

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For a subspace $S \subset \mathbb{F}_2^n$ and vectors $x \in \mathrm{co}(S), z \in \mathrm{co}(S^\perp)$, define

$$|S_{x,z}\rangle = \frac{1}{\sqrt{|S|}} \sum_{s \in S} (-1)^{s \cdot z} |s + x\rangle$$

General Recipe

$\mathrm{co}(S)$: a set of coset representatives of S

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$$\mathrm{H}^{\otimes n} \updownarrow$$

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Uncertainty principle: $\mathcal{A}(|S_{x,z}\rangle) \nRightarrow (s \in S + x \,, s' \in S^\perp + z)$

(if S, x, z are sufficiently random)

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Use x to hide the plaintext

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Use z as certificate of deletion

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- $\mathcal{H}$: family of hash functions
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Public-key encryption
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Instantiatiating the distribution over S

Optimize for…

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<u>Practicality</u>		

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[BI20]

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Publicly-Verifiable Deletion

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No entanglement required	Only two valid deletion certificates, so publish $\mathrm{OWF}(z), \mathrm{OWF}(z+v)$	
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[**B**KMPW23]

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[**B**KMPW23]

Publicly-Verifiable Ciphertext

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[**B**KMPW23]

Publicly-Verifiable Ciphertext

S uniform over all subspaces

$|S_{x,z}\rangle, \mathcal{C}_{sk}(S,h), b \oplus h(x)$

Instantiatiating the distribution over S

Optimize for…

Practicality

S spanned by standard basis vectors (Wiesner/BB84 states): $\theta \leftarrow \{0,1\}^n, S = \mathrm{span}\{e_i\}_{i:\theta_i=1}$

$\mathrm{H}^{\theta_1}|x_1\rangle, \ldots, \mathrm{H}^{\theta_n}|x_n\rangle,$
$\mathcal{C}_{sk}(\theta, h), b \oplus h(\{x_i\}_{i:\theta_i=0})$

No entanglement required

[BI20]

Publicly-Verifiable Deletion

S has dimension $n-1$, so $S^\perp = \{0^n, v\}$

$\mathrm{H}^{\otimes n}(|z\rangle + (-1)^x|z+v\rangle),$
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Practicality	Publicly-Verifiable Deletion	Publicly-Verifiable Ciphertext
S spanned by standard basis vectors (Wiesner/BB84 states): $\theta \leftarrow \{0,1\}^n, S = \text{span}\{e_i\}_{i:\theta_i=1}$	S has dimension $n-1$, so $S^\perp = \{0^n, v\}$	S uniform over all subspaces
$\text{H}^{\theta_1}\|x_1\rangle, \ldots, \text{H}^{\theta_n}\|x_n\rangle$, $\mathcal{C}_{sk}(\theta, h), b \oplus h(\{x_i\}_{i:\theta_i=0})$	$\text{H}^{\otimes n}(\|z\rangle + (-1)^x\|z+v\rangle)$, $\mathcal{C}_{sk}(v), b \oplus x$	$\|S_{x,z}\rangle, \mathcal{C}_{sk}(S,h), b \oplus h(x)$
No entanglement required	Only two valid deletion certificates, so publish $\text{OWF}(z), \text{OWF}(z+v)$	Secure even given oracle access to $S+x$
[BI20]	[**B**KMPW23]	[**B**GGKMRR23]

Outline

1. Basic scenario and applications
2. Recipe for constructions
3. Security
4. Certifiable deletion of programs

Security Game

Security Game

$\mathrm{CDExp}_{\mathcal{C},\mathcal{H},\mathcal{D},\mathcal{A}_1,\mathcal{A}_2}(b)$

- Sample $(S, x, z) \leftarrow \mathcal{D}$, $h \leftarrow \mathcal{H}$, and sk
- $\mathcal{A}_1\left(|S_{x,z}\rangle, \mathcal{C}_{sk}(S, h), b \oplus h(x)\right) \rightarrow \pi, \mathrm{st}$
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  - $\mathcal{C}$ one-time pad
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  - $\mathcal{C}$ subspace-hiding distribution
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Note: [Unruh 13] showed similar statement for a slightly different template supporting *quantum* certificates of deletion

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- Let $\mathcal{C}$ be a computationally-hiding statistically-binding commitment
- Let $\mathcal{H} = \bigoplus$ (unseeded)
- Let $\mathcal{D}$ sample a uniformly random (S, x, z)
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$\underline{\mathcal{A}}$ | $\mathrm{Hyb}_0(b)$ | $\underline{\mathit{Ch}}$

Sample (S, x, z)

$\leftarrow$ $\mathrm{Com}(S), b \bigoplus_i x_i, |S_{x,z}\rangle$

$\rightarrow$ π, st

If $\pi \notin S^\perp + z$, output $|\perp\rangle\langle\perp|$

Otherwise, output st

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Goal: Show that $\mathrm{TD}\big(\mathrm{Hyb}_0(0), \mathrm{Hyb}_0(1)\big) = \mathrm{negl}$

Example Proof

Hybrid 1: Delay the dependence of the experiment on b

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π, st →

If $\pi \notin S^\perp + z$, output $|\perp\rangle\langle\perp|$

If $b \bigoplus_i x_i \neq b'$, output $|\perp\rangle\langle\perp|$

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Example Proof

Hybrid 1: Delay the dependence of the experiment on b

$$\mathrm{TD}\big(\mathrm{Hyb}_1(0), \mathrm{Hyb}_1(1)\big) = \frac{1}{2}\mathrm{TD}\big(\mathrm{Hyb}_0(0), \mathrm{Hyb}_0(1)\big)$$

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Remains to show that x has a lot of conditional min-entropy

Example Proof

Want to show: If $\mathcal{A}(|S_{x,z}\rangle, \mathrm{Com}(S))$ outputs $\pi \in S^{\perp} + z$, then x has a lot of conditional min-entropy

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Claim: if $\mathcal{A}$ given random $v + z$ and outputs $\pi \in S^\perp + z$, then $\pi = v + z$ with overwhelming probability (over S, z)

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Measuring gives a uniformly random $x \in \mathrm{co}(S)$, *independent* of $\mathcal{A}$'s view

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Plan

- (Indistinguishability) obfuscation with certified deletion
- Applications
- Comparison with other notions

copy-protection

copy-detection

revocable crypto / key leasing

secure software leasing

Obfuscation with Certified Deletion

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Rough goal:

Obfuscation with Certified Deletion

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Candidate construction:

[**B**GGKMRR23]

$\left|S_{x,z}\right\rangle, \mathrm{CObf}(\mathrm{P}[S, f \oplus x])$

$\mathrm{P}\left[S, \tilde{f}\right](y, v)$:

- Let x be the coset of S that v belongs to
- Let $f = \tilde{f} \oplus x$
- Output $f(y)$

Obfuscation with Certified Deletion

A "one-way" compiler that scrambles the description of a circuit while maintaining its functionality

Rough goal:

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Candidate construction: [**B**GGKMRR23]

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- Before deletion, the program is useful in some way, after deletion it is not

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[**B**GGKMRR23]

$|S_{x,z}\rangle, \mathrm{CObf}(\mathrm{P}[S, f \oplus x])$

$\mathrm{P}[S, \tilde{f}](y, v)$:

- Let x be the coset of S that v belongs to
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- Output $f(y)$

Correctness: Given any input y, evaluate $\mathrm{Obf}(\mathrm{P}[S, f \oplus x])$ on y and in superposition over $S + x$ to learn $f(y)$

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If COb is modeled as a classical oracle:

- Before deletion, evaluator can use the oracle to learn $f(y)$ for any y of their choice
- After deletion (outputting $v \in S^{\perp} + z$), the evaluator cannot learn anything else from the oracle even given unbounded queries

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Seems like a weak guarantee, but (*differing inputs*) iO with CD are useful tools:

- Two-message delegation with certified deletion
- A generic compiler from encryption to encryption with *revocable secret keys*

Encryption with Revocable / Deletable Secret Keys

- $\text{Gen} \rightarrow \text{pk}, \text{vk}, |\text{sk}\rangle$
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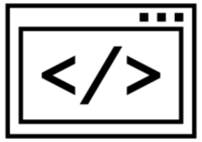
Privately-verifiable revocation from standard assumptions: [Kitagawa, Nishimaki 22], [Agarwal, Kitagawa, Nishimaki, Yamada, Yamakawa 23], [Ananth, Poremba, Vaikuntanathan 23]

Related Notions

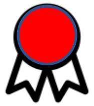
Hard for the adversary to produce…



“working” copy of a program

certificate derived from program

publicly verifiable



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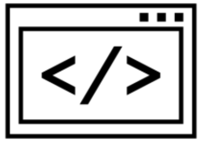
Copy Protection: [Aaronson 09]		



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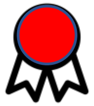

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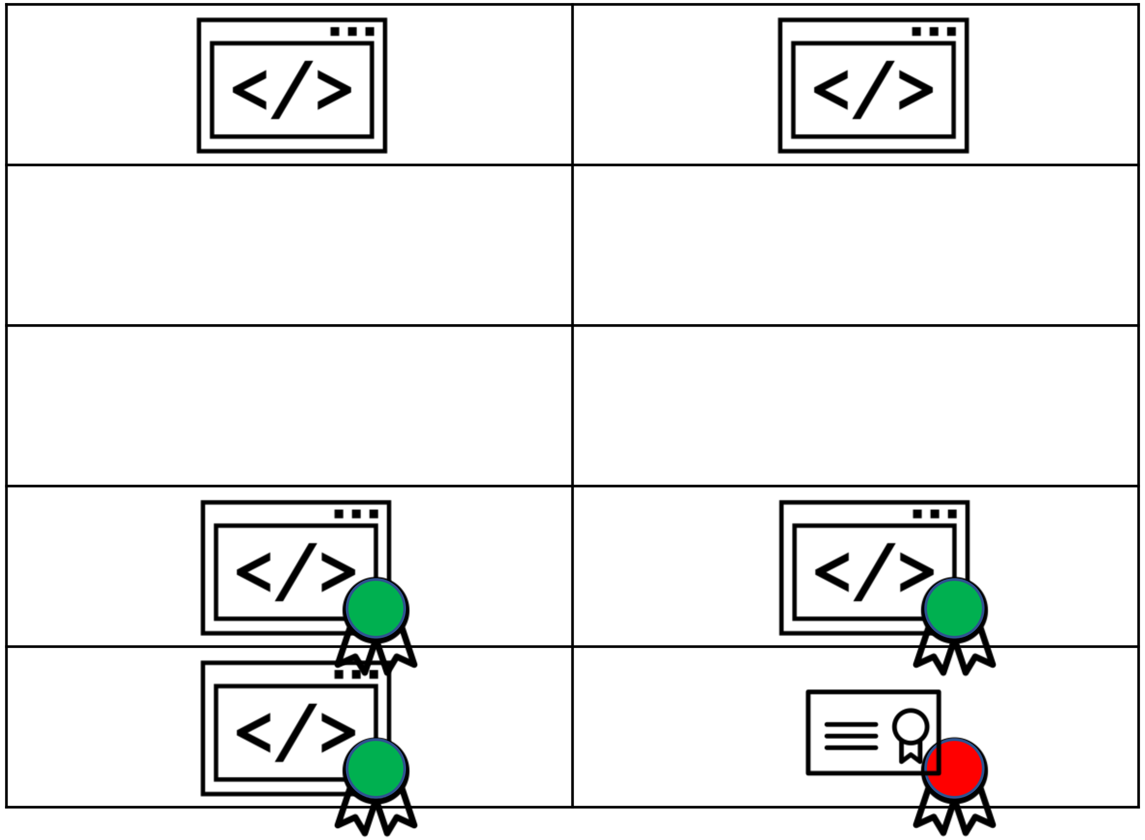

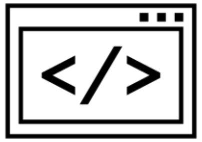
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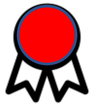
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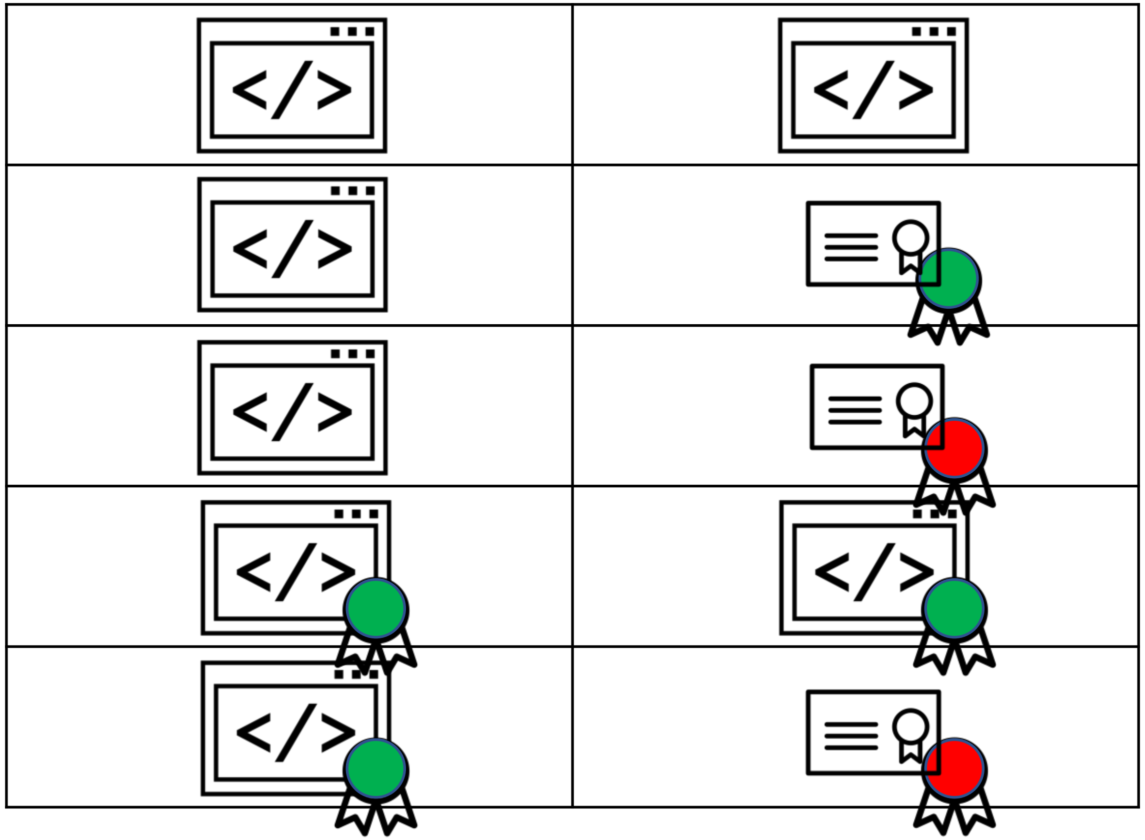

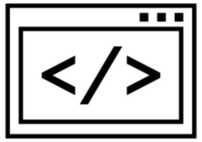 “working” copy of a program

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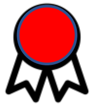 privately verifiable

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- More rigorous understanding of the relationship between unclonable primitives from previous slide ([Ananth, Kaleoglu, Liu 23])